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AGGREGATE VIEW FOR CAPACITATED LOT-SIZING

رؤية إجمالية لمشكلة تجزئ الطلب المُقيدة المصادر

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خلاصة:

يُعتبر تجزئ حجم الطلبية بأنواعه وتغيراته أحد القرارات الحرجة في تخطيط الإنتاج حيث أنه يعتمد على العديد من المتغيرات. و يهتم هذا البحث بالنوع الديناميكي، أحادى المرحلة، متعدد المنتجات، مُحدد السعة، مُتعدد الإعداد. يختص هذا النوع كلاسيكياً بتحديد خطة الإنتاج من حيث الكميات و التزامن خلال مجال زمني متقطع لمواكبة الطلب المتغير دورياً لجميع المنتجات بدون ترحيل، بحيث تتدنى التكاليف الكلية للإعداد والإنتاج والتخزين مع الوضع في الاعتبار أن جميع المنتجات مشتركة في مصدر وحيد محدود دورياً. و قد دلت البحوث السابقة على أن هذه المشكلة NP-hard من حيث الإتاحة والمثالية. لذلك يقترح هذا البحث أسلوب خاص بالمنتجات الصغيرة ذات الاستهلاك المحدود و المتقارب مع الأخذ في الاعتبار تكاليف الترحيل و الدونية. و تتأتى الفكرة الأساسية في تحويل المشكلة إلى توزيع المصادر المتاحة بشكل إجمالي بين الفترات ثم توزيعها بين المنتجات باستخدام نظام النقل ذا الكميات الثابتة في المرحلتين، وتتجزأ المرحلة الثانية إلى حل مشكلات منفصلة بعدد الفترات. و يتم هذا التحويل خلال بناء عدد متتالي من النماذج الرياضية.

ABSTRACT Lot-Sizing is a crucial decision arises, in many classes and variants, in the production planning. The paper focuses on the single-stage, dynamic, multi-item capacitated lot-sizing problem (CLSP) with continuous setups. It classically deals with the issue of determining a production plan for items in terms of quantities and their timing over a discrete finite horizon, so as to satisfy a known variable demand in each period without incurring backloggs and minimize the setup, production and holding costs. A finite production capacity of a single resource is shared by all items produced in each period. The literature showed that this problem is NP-hard in feasibilty and optimality. A special heuristic method is developed for small products. The mathematical models consider backorder and under capacity costs. The problem is reduced to a capacity distribution under special propositions and solved in two phases by using the fixed charge transportation problem (FCTP.) The first phase aggregately distributes the capacity between periods while the second decomposes this distribution into separate periods lot-sizing problems solvable as a sequence of FCTPs.

Keywords: Multi-Item; Dynamic Demand; Lot-sizing; Capacity Distribution; Decomposition

Introduction and Literature Review

Lot-Sizing problem—in its classes and variants—is defined by the nature of demand, available resources, number of items, production facility and planning

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time horizon. Two classes are common in practice, first class is known to be the economic lot-sizing and scheduling problem (ELSP) which models the situation of producing several items—with high demand, over a continuous infinite planning horizon, and under limited capacity. ELSP determines batching and sequencing of production items with or without setups in a single-stage or multi-stage facility (Elmaghraby, 1978; El-Najdawi, 1994; Gallego and Joneja, 1994; Bollapragada and Rao,1999; Federgruen and Katalan,1997.) The next class is known to be capacitated lot-sizing problem (CLSP) which suits dynamic demand over a finite discrete planning horizon. The production in each period is constrained by a finite capacity and no backorders are permitted. CLSP does not consider product sequencing and scheduling within a period. The reader may refer to Drexl and Kimms (1997) for the latter class and other lot-sizing problems. The CLSP arises in different variants and consisders, in most sectors, dynamic multi-item demand. Some of studies have taken into account setup time and cost in single-stage models (Trigeiro et al., 1989; Armentano et al., 1999; Vanderbeck, 1998) and others have implied negligible setups (Hindi, 1995.) Özdamar and Barbarosoğlu (1999) discussed the multi-item multi-stage CLSP and developed a hyprid model incorporates the loading issue; moreover, they mentioned other variants of the problem.

A quick review about pioneering work on the single stage problem with and without setup time can be found in Trigeiro et al. (1989). The CLSP considered here (§2) is a single-stage, dynamic, multi-item capacitated lot-sizing with continuous setups. It is concerned with finding the lot sizes of several products over a discrete finite horizon divided into T periods, so as to satisfy a dynamic demand in each period and minimize the costs of setup, production, inventory, backlogging and under-capacity. A specific production environment is assumed and a finite capacity is imposed for all products in each period. The setup is continuous in the sense that production occurs at any value up to capacity available in each period. A new heuristic methodology based on fixed charge transportation problem (FCTP) is developed. It reduces the problem, by using proposed parameters, to a capacity distribution solvable in two phases. The production capacity is distributed between periods in the first phase. The second phase decomposes into T single period sub-problems solvable as FCTPs. The solution of each sub-problem satisfies the demand of all products in each period through the distribution of the shared capacity. Each time a period is taken as demand period while the supply periods are predetermined from the first phase. The sequence in which periods are manipulated for demand is controlled by using some heuristic rules. This methodology is developed to accommodate a system having special nature for products and production processes. Analysis of the FCTP is not involved in the interest of this paper whearas it can be looked in Lamar and Wallace (1997), and Adlakha and Kowalski (1999).

The classical formulation of the problem is a mixed integer liner programming which minimizes the total costs of setup, production, and holding; without incurring backlogs (a model in Thizy and Van Wassenhove, 1985.) This problem is known to be NP-hard even for a single item. Such problem complexity is discussed in details in Chen and Thizy (1990). The solution of the problem includes optimal methods (Chen and Thizy, 1990) and heuristics

(Eisenhut, 1975; Dixon and Silver, 1981; Dogramaci et al., 1981; Thizy and VanWassenhove, 1985.) Because of complexity, most of research (heuristics and optimal methods) conducted some mathematical relaxations for the problem constraints and decomposed the problem into uncapacitated partial single-item sub-problems. The most often used approach to solve the problem optimally is to obtain a lower bound on its value—on the basis of generalized duality theory, by relaxing the capacity constraints—as a fathoming mechanism in an optimal enumerative search (Thizy and VanWassenhove, 1985.)

The partial Lagrangian relaxation of capacity constraints is a popular way results in a problem decomposes into a set of independent, uncapacitated, single product lot-sizing problems. The popular dynamic programming algorithm developed by Wagner and Whitin (1958) can be used to solve those sub-problems (Pan, 1994.) Thizy and VanWassenhove (1985) have proposed a method based on such relaxation and incorporated a primal partitationing scheme with a network flow problem to obtain some feasible solutions. They implemented the subgradient optimization algorithm and transportation problem to update the value of Lagrangian multipliers. Trigeiro et al. (1989) adopted this relaxation and subgradient optimization in their method to solve the problem with setup times. They followed a heuristic production smoothing procedure to generate feasible solutions.

Chen and Thizy (1990) have presented an attractive long analysis for the problem relaxation ways and relaxable components (binary variables, setup constraints, demand constraints and capacity constraints.) They focused on optimal methods and compared the various Lagrangian relaxations of the classical formulation especially the relaxation of the capacity constraints. They also discussed the linear programming relaxation and calculation of lower bounds via the known traditional algorithms: column generation; subgradient optimization; cutting planes and variables redefinition. Moreover, they analyzed other formulations (Eppen and Martin, 1987; Pochet and Wolsey, 1986) and mentioned the capability of some methods to accommodate setup times, backlogging and flexible capacities. Recently, Armentano et al. (1999) considered the setup time and cost and formulated the problem as a network flow model. A branch and bound method is proposed to solve the model where bounds are generated by the linear programming relaxation based on the capacity constraints. However, up to now, Lagrangian relaxation yields results better than linear programming relaxation.

Constantino (1998) discussed another concept for the problem relaxation—the polyhedra associated to the classical formulation with setup times and variable lower bound constraints. The presentad model is based on the model presented in Trigueiro et al. (1989). This concept seems to be not more than a theoritical analysis to the mixed integer models and therefore it can't be applied to the large scale problems.

However, as a result of the continuous change in the nature of products and markets, the problem size and parameters can't be assumed stable all times. Therefore verifying an optimal or even feasible solution is a difficult task and the results may be confusing when the problem is encountered in practice. It is proposed to adopt special heuristic methods accommodate the problem according to a classification for the production processes. For example, there is a tendency towards more flexible forms of production, which imply insignificant setups; thus making it possible to apply a method like what presented by Hindi (1997) in which the problem is formulated as a capacitated transhipment problem and solved as a simple network flow problem. Also, away from sophisticated mathemitical methods, an example for more simple and special method exists in Allen et al. (1997); the method is developed for schedulling a simple system—packaging lines at a major food manufacturer. Schedules were produces by using Excel and Visual Basic macros. Moreover, genetic algorithm is also used to simplify the solution of the problem with the most expected parameters (Hung et al., 1999.)

The multi-item CLSP in general seems to be a well analyzed problem. Therefore, concentrating attention have been to the previous work which was used as a guide for the special methodology developed in this paper. The next contributions include: summary of the assumptios which fit the special problem, design of symbols—parameters and decision varaibles—and developed functions (§2.2); formulation of the problem as a mixed integer linear programming (MILP) which modifies the classical formulation (§2.3); reduction and decomposition of the problem as a capacity distribution according to three propositions (§3.1); a solution procedure with a nemerical illustration of the final problem (§3.2); and conclusions (§4.)

Capacitated Lot-Sizing Configuration

Problem features

As declared before, this study is concerned with single-stage, dynamic, multi-item CLSP. The problem is developed through a sequence of mathematical models. The main features taken into account are summarized below:

1. Demands, capacity absorption rates, and costs of all products are known, independent, deterministic, and dynamic over a discrete planning horizon divided into independent periods. Note that the method is also valid for stochastic nature except demands;
2. Manufacturing process is regarded aggregately in a single stage as a common facility for all products, i.e. end product is the objective of lot-sizing;
3. A single global constrained (capacitated) resource is considered each time the problem is manipulated;
4. Capacity absorption rates of products are small and near in values;
5. Unit production costs are near for all products as well as holding and back-order costs;
6. Backlogging is permitted over the horizon except at the start and end, whereas safety stocks are not planned whenever. An upper bound could be imposed for backlogging in each period;
7. Production setups are not carried from period to the next;
8. Lots can't be split;
9. Resource capacity and its available expansion in each period is predetermined;
10. Producing a product in a period incurs a setup cost but with insignificant time. The setup is continuous, i.e. the production can take any value up to capacity once the facility is properly setup;

11.Lead time in each period is negligible and periods are not short;
12.Sequencing of products within a period is left for next production planning.

Terminilogy and nomenclatures

The problem parameters, functions and decision variables are defined according to their appearance in the paper as follows:

N number of products in the system;
T number of periods in the planning horizon;
S_{it} production setup cost of product i in period t;
d_{it} demand for product i in period t;
I_{it} on-hand inventory of product i at end of period t;
L_{it} backorder position of product i at end of period t;
x_{it} amount of product i produced in period t;
y_{it} binary variable has value 1 if $x_{it}>0$, 0 if $x_{it}=0$;
m sufficient large positive value; see mixed integer linear programming.
p_{it} unit production cost of product i in period t;
h_{it} unit holding cost of product i from period t to period $t+1$;
b_{it} unit backorder cost of product i from period t to period $t+1$;
u_t opportunity cost of losing a capacity unit;
a_{it} capacity consumed by each unit—absorption rate—of product i in period t;
C_t capacity of the constrained resource in period t;
C_{it}^{d} capacity associated with d_{it};
C_{it}^{x} capacity associated with x_{it};
C_{it}^{I} capacity associated with I_{it};
C_{it}^{L} capacity associated with L_{it};

$$\alpha_i = \sum_{t=1}^{T} a_{it} d_{it} / \sum_{t=1}^{T}\sum_{i=1}^{N} a_{it} d_{it} \qquad \forall i, \tag{i}$$

proposed system-weight ($0 \le \alpha_i \le 1$, $\sum_{i=1}^{N}\alpha_i = 1$) for product i, denotes its contribution, works as relative frequency;

$$\beta_t = \sum_{i=1}^{N} a_{it} d_{it} / \sum_{t=1}^{T}\sum_{i=1}^{N} a_{it} d_{it} \qquad \forall t, \tag{ii}$$

proposed system-weight ($0 \le \beta_t \le 1$, $\sum_{t=1}^{T}\beta_t = 1$) for period t, denotes its contribution, works as relative frequency;
w_h proposed weight for holding cost referring to the nature of products and storage facility;
w_b proposed weight for backorder cost referring to the nature of the market;

$$p_1 = \left(\sum_{t=1}^{T}\beta_t \sum_{i=1}^{N}\alpha_i \frac{p_{it}}{a_{it}}\right). \tag{iii}$$

expected internal production cost associated with each capacity unit;

$p_2 \quad = w_h(\sum_{t=1}^{T}\beta_t\sum_{i=1}^{N}\alpha_i\frac{h_{it}}{a_{it}})$, (iv)

per-period expected internal holding cost associated with each capacity unit;

$p_3 \quad = w_b(\sum_{t=1}^{T}\beta_t\sum_{i=1}^{N}\alpha_i\frac{b_{it}}{a_{it}})$, (v)

per-period expected internal backorder cost associated with each capacity unit;

S_t maximum total production setup cost in period t;

C_t^c total capacity consumed in period t;

C_t^I total capacity associated with inventory at end of period t;

C_t^L total capacity associated with backordering at end of period t;

S_{ij} pseudo setup cost assigned in period i (as supply) for period j (as demand);

C_{ij} capacity transported between periods i (as supply) and j (as demand);

y_{ij} binary variable has value 1 if C_{ij}>0, 0 if C_{ij}=0;

$$p_{ij} = \begin{cases} p_1 + p_2 & \text{if } i < j \\ p_1 & \text{if } i = j \\ p_1 + p_3 & \text{if } i > j \end{cases} \quad \forall i, j = 1, 2, \ldots T, \qquad \text{(vi)}$$

expected internal transportation cost between periods i (as supply) and j (as demand) for each capacity unit;

$$p_{i\tau t} = \begin{cases} (p_{i\tau} + \sum_{k=\tau}^{t-1} h_{ik})/a_{i\tau} & \text{if } \tau < t \\ p_{it}/a_{i\tau} & \text{if } \tau = t \\ (p_{i\tau} + \sum_{k=t}^{\tau-1} b_{ik})/a_{i\tau} & \text{if } \tau > t \end{cases} \quad \forall i; \tau \in H_t; t, \qquad \text{(vii)}$$

unit transportation cost of product i between periods τ (as supply) and t (as demand);

$C_{i\tau t}$ capacity consumed in period τ to satisfy a part of demand of product i in period t;

H set of periods in the time horizon;

H_t set of periods having the production which satisfies demand of period t;

$C_{\tau t}$ capacity consumed in period τ to satisfy a part of demand (mixed) in period t;

$y_{i\tau t}$ binary variable has value 1 if $C_{i\tau t}$ >0, 0 if $C_{i\tau t}$ =0.

These notations will be followed hence and forth, without further details, in modeling, relaxing, decomposing, and solving CLSP (§2.3, §3.1, and §3.2.) .

Original formulation of the problem

The classical formulation is adopted (Thizy and VanWassenhove, 1985) and extended to accommodate backlogging and under-capacity cost. Hence, the single-stage, dynamic, multi-item CLSP with continuous setups can be formulated as follows.

$$P_1: Min\ Z = \sum_{t=1}^{T}\sum_{i=1}^{N}(y_{it}S_{it} + p_{it}x_{it} + h_{it}I_{it} + b_{it}L_{it}) + \sum_{t=1}^{T}u_t(C_t - \sum_{i=1}^{N}a_{it}x_{it}) \quad (1)$$

subject to $d_{it} = x_{it} - I_{it} + I_{i(t-1)} + L_{it} - L_{i(t-1)} \quad \forall i,t$ (2)

$$\sum_{i=1}^{N} a_{it}x_{it} \le C_t \quad \forall t \quad (3)$$

$$x_{it} \le my_{it} \quad \forall i,t \quad (4)$$

$$y_{it} \in \{0,1\} \quad \forall i,t \quad (5)$$

$$L_{i0}, L_{iT} = 0 \quad \forall i \quad (6)$$

$$x_{it}, I_{it}, L_{it} \ge 0 \quad \forall i,t \quad (7)$$

The objective function (1) minimizes the total costs of setup, production, holding, backlogging, and under-capacity. Constraint (2) includes the flow balance between demand, production, on-hand inventory, and backorders. Constraint (3) limits production up to an available capacity and (4)&(5) assures that setup cost is incurred only when a product is produced in a period (m is a very large positive value.) Without loss of generality, initial and final backlogging are set to zero (constraint (6).) Constraint (7) is set for nonnegativity of production, on-hand inventory and backorders.

Capacity Manipulation of the Problem

Relaxed formulations and decomposition

Referring to problem P_1, the variables represent number of units can be easily replaced with the equivalent capacities. Then the objective function (1) can be written as

$$P_1: Min\ Z = \sum_{t=1}^{T}\sum_{i=1}^{N}(y_{it}S_{it} + \frac{p_{it}}{a_{it}}C_{it}^{x} + \frac{h_{it}}{a_{it}}C_{it}^{I} + \frac{b_{it}}{a_{it}}C_{it}^{L}) + \sum_{t=1}^{T}u_t(C_t - \sum_{i=1}^{N}C_{it}^{x}) \quad (8)$$

while the problem constraints are modified as shown in (10 to 15.) The next two propositions in addition to the cost functions developed in §2.2 modifies problem P_1 to a complete capacity form as declared in model (9 to 15.)

PROPOSITION 1—Suppose that a large CLSP, with a single capacity constraint, arises with products having consumption rates a_{it} vary in a specific pattern (determined by a frequency distribution for NT values) can be represented by a single random variable with estimable mean and standard deviation. Thus making p_{it}/a_{it}, h_{it}/a_{it} and b_{it}/a_{it} can be represented by independent random variables with estimable means p_1, p_2 and p_3 and standard deviations σ_1, σ_2 and σ_3 respectively even if the numerators are not random variables.

PROPOSITION 2—Based on proposition 1, if mean and standard deviation of a_{it} is small (e.g., for instance of time capacity it rates in minutes), the problem can be approximated by capacity distribution problem P_2 which could be reduced to problem P_3. Then

$$P_2: \; Min \; Z = \sum_{t=1}^{T}\sum_{i=1}^{N}(y_{it}S_{it} + p_1C_{it}^{x} + p_2C_{it}^{I} + p_3C_{it}^{L}) + \sum_{t=1}^{T}u_t(C_t - \sum_{i=1}^{N}C_{it}^{x}) \tag{9}$$

subject to
$$C_{it}^{d} = C_{it}^{x} - C_{it}^{I} + C_{i(t-1)}^{I} + C_{it}^{L} - C_{i(t-1)}^{L} \quad \forall i,t \tag{10}$$

$$\sum_{i=1}^{N}C_{it}^{x} \le C_t \quad \forall t \tag{11}$$

$$C_{it}^{x} \le my_{it} \quad \forall i,t \tag{12}$$

$$y_{it} \in \{0,1\} \quad \forall i,t \tag{13}$$

$$C_{i0}^{L}, C_{iT}^{L} = 0 \quad \forall i,t \tag{14}$$

$$C_{it}^{x}, C_{it}^{I}, C_{it}^{L} \ge 0 \quad \forall i,t \tag{15}$$

Substituting with the cumulative quantities changes the model P_2 to

$$P_3: \; Min \; Z = \sum_{t=1}^{T}(S_t + p_1C_t^{x} + p_2C_t^{I} + p_3C_t^{L}) + \sum_{t=1}^{T}u_t(C_t - C_t^{x}) \tag{16}$$

subject to
$$C_t^{x} \le C_t \quad \forall t \tag{17}$$

$$S_t \le \sum_{i=1}^{N}S_{it} \quad \forall t \tag{18}$$

$$S_t, C_t^{x}, C_t^{I}, C_t^{L} \ge 0 \quad \forall t \tag{19}$$

PROPOSITION 3—The problem P_2 can be decomposed into two phases of FCTPs such as problem $P_{3\text{-}1}$ (20 to 25) and problem $P_{3\text{-}2}$ (26 to 32). if the setup costs are continuous, small, and not significantly different. Problem $P_{3\text{-}1}$ represents the first phase in which the available periodical capacities will be distributed between the horizon periods according to demands. Problem $P_{3\text{-}2}$ represents the second phase which in turn decomposes into T sub-problems to distribute each period capacity—assigned in the first phase—between the products. Hence, the final plan appears easily as product quantities produced, stored, and backordered in each period.

First Phase Problem: P_{3-1} : $Min\ Z_1 = \sum_{i=1}^{T}\sum_{j=1}^{T}(y_{ij}S_{ij} + p_{ij}C_{ij})$ (20)

subject to

$$\sum_{j=1}^{T} C_{ij} \le C_i \qquad \forall i = 1, 2, ..., T \tag{21}$$

$$\sum_{i=1}^{T} C_{ij} \le \sum_{k=1}^{N} a_{kj} d_{kj} \qquad \forall j = 1, 2, ..., T \tag{22}$$

$$C_{ij} \le m y_{ij} \qquad \forall i, j = 1, 2, ..., T \tag{23}$$

$$y_{ij} \in \{0,1\} \qquad \forall i, j = 1, 2, ..., T \tag{24}$$

$$C_{ij} \ge 0 \qquad \forall i, j = 1, 2, ..., T \tag{25}$$

Second Phase Problem: P_{3-2} : $Min\ Z_2 = \sum_{t\in H}\ \sum_{\tau\in H_t}\ \sum_{i=1}^{N}(y_{i\tau t}S_{i\tau} + p_{i\tau t}C_{i\tau t}), S_{i\tau} \in S_{it}$

(26)

subject to

$$C_{i\tau t} \le m y_{i\tau t} \qquad \forall i; \tau \in H_t; t \tag{27}$$

$$y_{i\tau t} \in \{0,1\} \qquad \forall i; \tau \in H_t; t \tag{28}$$

$$\sum_{t\in H} y_{i\tau t} = 1 \qquad \forall i; \tau \in H_t \tag{29}$$

$$\sum_{i=1}^{N} C_{i\tau t} = C_{\tau t} \qquad \forall \tau \in H_t; t \tag{30}$$

$$\sum_{\tau\in H_t} \frac{C_{i\tau t}}{a_{i\tau}} \le d_{it} \qquad \forall i, t \tag{31}$$

$$C_{i\tau t} \ge 0 \qquad \forall i; \tau \in H_t; t \tag{32}$$

Solution procedure and numerical illustration

Table I—Appendix shows the data of a typical problem has six production-distribution periods (T=6.) The aspired plan is capacitated with time—a common single constraint problem. The proposed system dimensionless weights α_t and β_t were calculated and checked in this table. Internal cost parameters per unit capacity p_1, p_2, and p_3 were calculated in Tables II, III, and IV—Appendix respectively. As mentioned before, the problem is reduced to a decomposed FCTP problem explained by two formulations $P_{3\text{-}1}$ & $P_{3\text{-}2}$. Therefore, the solution procedure includes two phases. The first phase distributes the capacities between periods using the FCTP (see Adlakha and Kowalski, 1999 to know more about FCTP) which is tabulated as shown in Table V—Appendix of the numerical example. The value of S_{ij} is not a real setup but it is set to avoid the extra partitioning of the available capacities; it could be experimented over a range of values less than S_t or set as being per unit capacity. After constructing and solving Table V, the procedure transfers to the next phase. The first phase model of the current example is solved using an LP Software which results a time distribution as shown in Table VI—Appendix. The second phase solves T FCTPs, each of them can be tabulated in a sheet similar to Table VII—Appendix. Each table considers a period for demand and distributes the assigned capacities between the products existing in that period. The salient question now is what is the order in which the T periods are manipulated and what is the effect of this order? It is recommended to rank each period order according to: $\Sigma a_{it}d_{it}$; $\Sigma p_{it}+\Sigma\Sigma(h_{it})$; $\Sigma\Sigma(b_{it})$ and ΣS_{it}. Hence, the average rank is followed to order the periods in an ascending order. Otherwise, all orders could be tried. If a product is assigned to a period for production, a zero setup cost is assigned to this product in this period in the next step.

The Spreadsheet of *MS-Excel* can be adopted as a calculation engine besides any LP Software. Note that the cited tables could be used to summarize the problem and they don't need to further explanation. Also, the guiding heuristic rules may be slightly modified by using for instance some statistical quantities.

Discussion and Conclusions

A special purpose heuristic methodology was developed to solve the single constraint, single-stage, dynamic, multi-item CLSP with continuous setups. It is particularly designed for planning a special class of products—coalesced products and small discrete products, which are produced in large quantities. The objective is to set a production plan in terms of quantities and timing at the possible minimum sum of the setup, production, holding, backorder, and under capacity costs without overtime. The methodology consists of two phases based on distributing the capacities between the periods, and hence between the products in each period instead of distributing the products themselves. First solves one FCTP and second solves a sequence of FCTPs. A set of weighting parameters was proposed to relax and decompose the problem into this form which is found amenable than those in the literature. The routine of formulations starts with the known classical MILP formulation. The literature proved that this problem is NP-hard even for a single item (Chen and Thizy, 1990.) Therefore, several researches confirmed the essence of developing special methods to accommodate real industrial cases (Allen et al., 1997.)

As a matter of fact, the cost structures are never really known in advance and the learning factor can't be neglected. Therefore, those structures were dealt with as if they are stochastic. For instance, the cost parameters p_1, p_2 and p_3—stated in proposition 1—reflect this assumption which can be realized if their variances would be small. Thus making the proposed methodology performs quite well in the cases of coalesced products—pharmaceuticals, chemical fluids and powders, metal sheets, etc. However, this paper demonstrates a new view for the problem that aggregates and distributes capacities of a single resource, although it may lead to some unexplained insignificant losses.

The findings indicate that the setups is the unique parameter which may limit the methodology applications and extensions. Therefore, it may reach about 100% if the setups approaches zero and absorption rates become smaller. It is interesting that this methodology seems to be more efficient in handling the very large demand problems better than those small demand problems. We finish by listing topics for further extensions: using additional constraints (Glover and Ross,1974); merging the more-for-less paradox in case of increasing demands (Adlakha and Kowalski, 1998); using the concept of multi-commodity (Evans and Jarvis, 1977); and sequencing the products in each period.

However, further work remains to be done on such CLSPs because no known method guarantees optimal or even feasible solutions. In other words, it can be said that it is difficult to include and manipulate all parameters in the same way for such NP-hard problems. Therefore, the developed work follows the tendency towards providing special methods which accommodate a limited range of problems like those which imply limited setups, eonsumption rates of capacity, and/or costs. Moreover, the proposed method can implement the problems of stochastic nature except demand.

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Appendix: Numerical summary of the solution procedure

This appendix comprises seven tables demonstrate the steps of the solution procedure by using numerical data of four products. The first table exhibts the original data of the four products (demand per period; unit costs of production, holding, and backorder; and absorbtion rate of the time resource) in addition to the primary calculations of the proposed parameters. The second. third and fourth tables exhibt the calculations of the internal costs (production. holding and backorder.) Table five demonestrates the first phase FCTP distribution. Table six exhibts the output of an LP Software soltion of the fifth table. Table seven demonstrates conducting the second phase to a sample period (sample of six tables)—distributing the capacity between each period as a source and the six periods as destinations.

Shift time 8.0 hours

Hours available per day based on 2.25 shifts 18.0 hours

	Product Demands (10^3) for Six Periods						Setup $	Inventory Costs			Rate	Total	Product Weighting			
Product	1	2	3	4	5	6	S	p	h	b	a	Rate	α	$\alpha(p/a)$	$\alpha(h/a)$	$\alpha(b/a)$
1	2.0	1.5	2.5	0.0	3.0	3.5	300	3.0	0.3	0.5	3.0	660.000	0.27481	0.27481	0.02748	0.04580
2	3.0	2.0	0.0	4.0	1.0	2.5	350	2.5	0.2	0.5	2.5	561.458	0.23378	0.23378	0.01870	0.04676
3	2.0	3.0	2.5	4.0	2.0	1.0	320	2.0	0.4	0.5	2.5	656.250	0.27325	0.21860	0.04372	0.05465
4	2.0	0.0	1.5	2.0	5.0	1.5	300	3.0	0.5	0.5	2.5	523.958	0.21816	0.26180	0.04363	0.04363
Period Index ν	1.20	1.15	1.10	1.05	1.00	0.95	Average Learning = 95.5%					Total = 2401.67	1.000	0.98898	0.13354	0.19084
Req. Time (hrs)	470.000	325.833	320.833	437.500	483.333	364.167	Total = 2401.67 hours					hours		$per min.	$per min.	$per min.
Req. Time (days)	26.1111	18.1019	17.8241	24.3056	26.8519	20.2315	Total = 133.426 days							p_1	p_2	p_3
Period Weight β	0.19570	0.13567	0.13359	0.18217	0.20125	0.15163	Total = 1.00							Non-completely indexed		
Capacity (days)	23.0	25.0	22.0	22.0	22.0	22.0	ν is an index used for periodical variation of capacity absorption rate									
Capacity (hrs)	414.000	450.000	396.000	396.000	396.000	396.000	a is a base for capacity absorption rate (minutes per unit)									
Slack (days)	-3.1111	6.8981	4.1759	-2.3056	-4.8519	1.7685	Total = 2.57407 days									
Product Averages: Costs; Rate; Weight; and Weighted Costs							317.500	2.6250	0.3500	0.5000	2.625	600.417	0.25000	0.24725	0.03338	0.04771
Product Stds.: Costs; Rate; Weight; and Weighted Costs							23.6291	0.4787	0.1291	0.0000	0.2500	68.3890	0.0285	0.0256	0.0124	0.0048

Table I. Data of products: demand; production, holding, and backorder costs; absorption rates; and calculations of period & product weights.

Cost 1	Six Periods					
Product	1	2	3	4	5	6
1	0.22901	0.23896	0.24983	0.26172	0.27481	0.28927
2	0.19482	0.20329	0.21253	0.22265	0.23378	0.24608
3	0.18217	0.19009	0.19873	0.20819	0.21860	0.23010
4	0.21816	0.22765	0.23800	0.24933	0.26180	0.27558
$\sum\alpha(p/av)$	0.82415	0.85999	0.89908	0.94189	0.98898	1.04104
$\beta\sum\alpha(p/av)$	0.16128	0.11667	0.12011	0.17158	0.19903	0.15785
$\sum\beta\sum\alpha(p/av)$	0.92653	*expected internal-production cost/min.*				

Table II. Production cost calculations.

Cost 2	Six Periods					
Product	1	2	3	4	5	6
1	0.02290	0.02390	0.02498	0.02617	0.02748	0.02893
2	0.01559	0.01626	0.01700	0.01781	0.01870	0.01969
3	0.03643	0.03802	0.03975	0.04164	0.04372	0.04602
4	0.03636	0.03794	0.03967	0.04156	0.04363	0.04593
$\sum\alpha(h/av)$	0.11128	0.11612	0.12140	0.12718	0.13354	0.14056
$\beta\sum\alpha(h/av)$	0.02178	0.01575	0.01622	0.02317	0.02687	0.02131
$\sum\beta\sum\alpha(h/av)$	0.12510	*expected internal-holding cost/min.*				

Table III. Holding cost calculations.

Cost 3	Six Periods					
Product	1	2	3	4	5	6
1	0.03817	0.03983	0.04164	0.04362	0.04580	0.04821
2	0.03896	0.04066	0.04251	0.04453	0.04676	0.04922
3	0.04554	0.04752	0.04968	0.05205	0.05465	0.05753
4	0.03636	0.03794	0.03967	0.04156	0.04363	0.04593
$\sum\alpha(b/av)$	0.15903	0.16595	0.17349	0.18175	0.19084	0.20088
$\beta\sum\alpha(b/av)$	0.03112	0.02251	0.02318	0.03311	0.03841	0.03046
$\sum\beta\sum\alpha(b/av)$	0.17879	*expected internal-backorder cost/min.*				

Table IV. Backorder cost calculations.

Periods (i, j)	1	2	3	4	5	6	Available	
1	0.9265	1.0516	1.1767	1.3018	1.4269	1.5520	23	24840
2	1.1053	0.9265	1.0516	1.1767	1.3018	1.4269	25	27000
3	1.2841	1.1053	0.9265	1.0516	1.1767	1.3018	22	23760
4	1.4629	1.2841	1.1053	0.9265	1.0516	1.1767	22	23760
5	1.6417	1.4629	1.2841	1.1053	0.9265	1.0516	22	23760
6	1.8205	1.6417	1.4629	1.2841	1.1053	0.9265	22	23760
Required (days)	26.111	18.102	17.824	24.306	26.852	20.231		146880
Required(mins.)	28200	19550	19250	26250	29000	21850	144100	-2780
Internal Costs	p_1= 0.9265		p_2= 0.1251		p_3= 0.1788		$ per min., S_{ij}=0	

Table V. FCTP in first phase: costs p_{ij} in $ per minute and S_{ij} in $.

Periods as Sources	Periods as Destinations 1	2	3	4	5	6	Unused Time
1	24840	...	...	...	...	...	...
2	3360	19550	1310	...	...	...	2780
3	...	...	17940	5820	...	...	...
4	...	...	...	20430	3330	...	...
5	...	...	...	...	23760	...	...
6	...	...	...	...	...	21850	...

Table VI. Time distribution (minutes) between periods.

Assignment Order No. (1)	Period No. (1) as a Destination						Required
	Periods as Sources						Time
Poduct	1	2	3	4	5	6	in mins.
1 (p)	3.0	3.0	M	M	M	M	7200
S	300.0	300.0	M	M	M	M	
2 (p)	2.5	2.5	M	M	M	M	9000
S	350.0	350.0	M	M	M	M	
3 (p)	2.0	2.0	M	M	M	M	6000
S	320.0	320.0	M	M	M	M	
4 (p)	3.0	3.0	M	M	M	M	6000
S	300.0	300.0	M	M	M	M	
Available (mins.)	24840	3360	0	0	0	0	28200

Costs: S_{it} in $ and p_{i-t} in $ per minute (capacity unit).

Table VII. FCTPs of the second phase: a sample table of six tables.

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